

Solar System limits on gravitational dipoles

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ABSTRACT

The gravitational dipole theory of Hadjukovic (2010) is based on the hypothesis that antimatter has a negative gravitational mass and thus falls upwards on the Earth. Astrophysically, the model is similar to but more fundamental than Modified Newtonian Dynamics (MOND), with the Newtonian gravity g_N towards an isolated point mass boosted by the factor $\nu = 1 + (\alpha/x) \tanh(\sqrt{x}/\alpha)$, where $x \equiv g_N/a_0$ and $a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}$ is the MOND acceleration constant. We show that α must lie in the range 0.4–1 to acceptably fit galaxy rotation curves. In the Solar System, this interpolating function implies an extra Sunwards acceleration of αa_0 . This would cause Saturn to deviate from Newtonian expectations by $7000(\alpha/0.4) \text{ km}$ over 15 yr, starting from known initial position and velocity on a near-circular orbit. We demonstrate that this prediction should not be significantly altered by the postulated dipole haloes of other planets due to the rather small region in which each planet’s gravity dominates over that of the Sun. The orbit of Saturn should similarly be little affected by a possible ninth planet in the outer Solar System and by the Galactic gravity causing a non-spherical distribution of gravitational dipoles several kAU from the Sun. Radio tracking of the Cassini spacecraft orbiting Saturn yields a 5σ upper limit of 160 m on deviations from its conventionally calculated trajectory. These measurements imply a much more stringent upper limit on α than the minimum required for consistency with rotation curve data. Therefore, no value of α can simultaneously match all available constraints, falsifying the gravitational dipole theory in its current form at extremely high significance.

Key words: gravitation – space vehicles – celestial mechanics – ephemerides – solar neighbourhood – dark matter.

1 INTRODUCTION

One of the great mysteries in contemporary astronomy is the true cause of the very large dynamical discrepancies between the observed rotation curves of galaxies and the predictions of Newtonian gravity applied to their luminous matter distributions (e.g. Babcock 1939; Rubin & Ford 1970; Rogstad & Shostak 1972). These acceleration discrepancies are usually attributed to dark matter haloes surrounding each galaxy (Ostriker & Peebles 1973). This interpretation is challenged by continued null detection of the constituent dark matter particles in sensitive searches, for instance in 11 yr of *Fermi* data on dwarf spheroidal satellites of the Milky Way (Hoof, Geringer-Sameth & Trotta 2020). Moreover, the galactic acceleration discrepancies follow some remarkable regularities (Famaey & McGaugh 2012) that can be summarized as a unique relation between the acceleration inferred from the rotation

curve and that expected from the baryonic distribution (McGaugh, Lelli & Schombert 2016).

Such a radial acceleration relation (RAR) was predicted several decades earlier using Modified Newtonian Dynamics (MOND, Milgrom 1983). In MOND, the supposed dynamical effect of dark matter is instead provided by an acceleration dependence of the gravity law. The gravitational field strength g at distance r from an isolated point mass M transitions from the Newtonian GM/r^2 law at short range to

$$g = \frac{\sqrt{GMa_0}}{r} \quad \text{for } r \gg r_M \equiv \sqrt{\frac{GM}{a_0}}. \quad (1)$$

MOND introduces a_0 as a fundamental acceleration scale of nature below which the deviation from Newtonian dynamics becomes significant. For a point mass, this corresponds to distances beyond its MOND radius r_M .

Empirically, $a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}$ to match galaxy rotation curves (Begeman, Broeils & Sanders 1991; McGaugh 2011). Remarkably, this is the same order of magnitude as the acceleration at which the classical energy density of a gravitational field (equation

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9 of Peters 1981) becomes comparable to the dark energy density $u_\Lambda \equiv \rho_\Lambda c^2$ that conventionally explains the accelerating expansion of the Universe (Efstathiou, Sutherland & Maddox 1990; Ostriker & Steinhardt 1995; Riess et al. 1998).

$$\frac{g^2}{8\pi G} < u_\Lambda \Leftrightarrow g \lesssim 2\pi a_0. \quad (2)$$

MOND could thus be a result of poorly understood quantum gravity effects that also underlie dark energy (e.g. Milgrom 1999; Pazy 2013; Verlinde 2016; Smolin 2017).

Regardless of the microphysical explanation, MOND can accurately match the rotation curves of a wide variety of spiral and elliptical galaxies across a vast range in mass, surface brightness, and gas fraction using only their luminous matter (fig. 5 of Lelli et al. 2017). This only became apparent long after the MOND field equation was first published (Bekenstein & Milgrom 1984), making these achievements successful a priori predictions. It is difficult to explain the success of these predictions in a conventional gravity context, even with the observational facts in hand (Desmond 2017a, b; Ghari et al. 2019). In particular, the latter work showed that it is still very difficult to keep a tight RAR despite a diversity of rotation curve shapes at a fixed peak velocity (Oman et al. 2015).

Despite its successes, the empirical MOND approach lacks a firm theoretical foundation. The gravitational dipole (GD) model seeks to explain the MOND phenomenology using more fundamental concepts (Hajdukovic 2010). In this model, antimatter particles have a negative gravitational mass with the same magnitude as the corresponding particle. The gravitational properties of antimatter are under investigation at CERN thanks to the experiments known as GBAR (Pérez et al. 2015), AEgIS (Brusa et al. 2017), and ALPHA-g (Bertsche 2018). Experimental uncertainties are currently $\approx 100\times$ too large to test the hypothesis that antimatter falls upwards in the terrestrial gravitational field (Alpha Collaboration 2013). Decisive results should be available by the end of 2021.

If antimatter antigravitates in the sense described, matter–antimatter pairs resulting from quantum vacuum (QV) fluctuations would be virtual GDs. An important consequence is that the QV contains no net gravitational mass, leading to a predicted cosmological constant of zero. In the GD model, the dynamical discrepancies in galaxies are an effect of gravitational polarization of the GDs. The divergence of this polarized field of GDs leads to extra effective mass, similarly to polarization in electrostatics (equation 12).

In this contribution, we focus on what the GD model implies for the Solar System. The accurate data within it place tight constraints on modified gravity theories (Hees et al. 2016). In the context of MOND as modified gravity, this is mainly due to the Galactic external field effect (Milgrom 1986) causing the Solar ‘phantom’ dark matter halo to be non-spherical at large distances. Thus, even without any phantom dark matter within the inner Solar System, deviations from Keplerian motion are generically expected due to a non-zero tidal stress (Blanchet & Novak 2011).

The GD model originally predicted a fixed extra Sunwards acceleration of $\approx 7a_0$ in the Solar System (equation 12 of Hajdukovic 2010). This was in line with the ‘Pioneer anomaly’, a then-unexplained Sunwards acceleration of the Pioneer spacecraft (Turyshev & Toth 2010). However, this anomaly is likely caused by thermal forces (Turyshev et al. 2012). The expected anomalous acceleration was later revised down to a_0 (Hajdukovic 2012) and then to the range $(0.1\text{--}1)a_0$ (equation 6 of Hajdukovic 2013). A theoretically motivated interpolating function was then provided between the ‘saturated’ (high-acceleration) region near a massive

body and the low-acceleration region further away, where the extra gravity of the GDs is expected to dominate (section 2 of Hajdukovic 2014b). We make use of equation (2) in that work, which was also restated in equation (24) of Hajdukovic (2014a). These equations are analogous to our equation (3). Importantly for our work, the latest publication advocating the GD model predicts that Solar System planets should experience an extra Sunwards acceleration of $(0.4\text{--}0.5)a_0$, with the most likely value being $5a_0/12$ (section 6.1 of Hajdukovic 2020).

The effects of radiation pressure can be minimized by accurately tracking the motion of a gas giant planet, as done by the Cassini mission (Matson 1992). Using radio tracking data from orbit around Saturn, Hees et al. (2014) constrained the tidal stress on the Solar System to be $(3 \pm 3) \times 10^{-27} \text{ s}^{-2}$ in the direction towards the Galactic Centre (see their equation 6). At the 9.58 AU distance of Saturn, this corresponds to an acceleration of $(4.3 \pm 4.3) \times 10^{-15} \text{ m s}^{-2}$. While this constraint is not directly applicable to an additional Sunwards force, it does suggest that any such anomalous acceleration should be $\lesssim 10^{-4}a_0$. This casts serious doubt on the GD model (Iorio 2019). We investigate the model in more detail to see if there is any way around this apparent falsification.

After introducing the GD model and its context (Section 1), we explore what a GD force law consistent with galaxy rotation curves (Section 2) implies for the motion of a planet in the Solar System (Section 3). We find that the GD model predicts very large deviations (Sections 3.1–3.2) that greatly exceed observational upper limits (Section 3.3), as previously found by Iorio (2019). We then explore whether this conclusion can be weakened once we take into account the gravity of other planets and their dipole haloes (Section 3.4), the possible presence of a ninth planet in the outer Solar System (Section 3.5), and the Galactic gravity (Section 3.6). Our conclusions are given in Section 4.

2 GALAXY ROTATION CURVE CONSTRAINT

In this section, we consider galaxy rotation curve constraints on the isolated point mass force law in the GD model, which can be expressed as (equation 24 of Hajdukovic 2014a):

$$g = g_N + \alpha a_0 \tanh\left(\frac{r_M}{\alpha r}\right). \quad (3)$$

This reduces to Newtonian dynamics when the Newtonian gravity $g_N \equiv GM/r^2 \gg a_0$. It also reproduces the correct asymptotic behaviour for $g_N \ll a_0$ (equation 1).

Equation (3) can be written in the standard MOND terminology with interpolating function

$$\nu_{\text{GD}} = 1 + \frac{\alpha}{x} \tanh\left(\frac{\sqrt{x}}{\alpha}\right), \quad (4)$$

where $x \equiv g_N/a_0$ and the actual gravity $g \equiv \nu g_N$, with ν being the factor by which a model enhances the strength of gravity towards an isolated point mass. The subscript ‘GD’ indicates that equation (4) is motivated by the GD model.

The ν function is constrained by the baryonic mass distributions and rotation curves of disc galaxies, with the former determining g_N while the latter measures the true radial gravity g . The empirical relation between g and g_N (the RAR) can be fitted very well by equation (11) of Lelli et al. (2017):

$$\nu_{\text{RAR}} = \frac{1}{1 - e^{-\sqrt{x}}}. \quad (5)$$

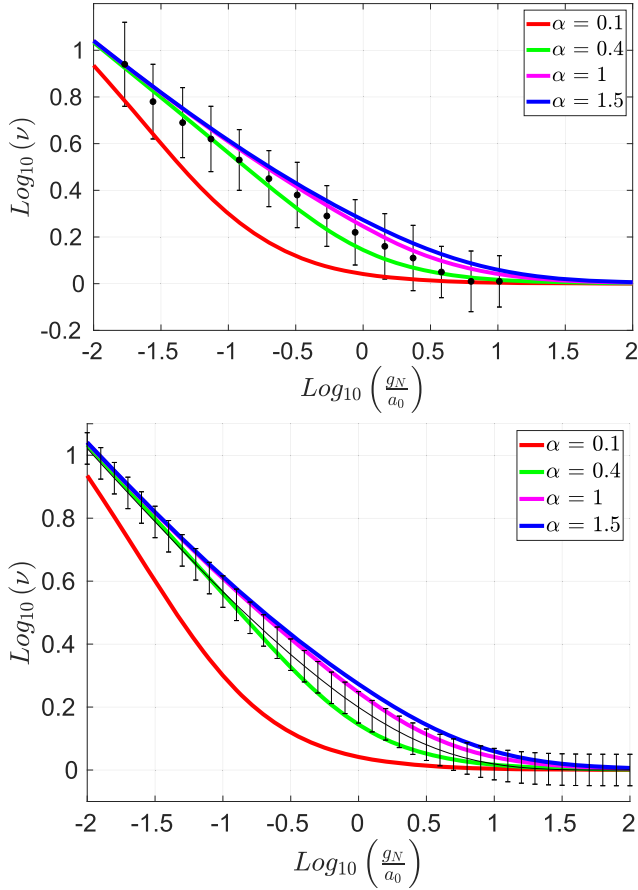


Figure 1. Dipole dark matter-based interpolating functions (equation 4) for different values of α (curves with higher α have higher v). We also show *top*: the binned RAR data from Lelli et al. (2017) with error bars showing (conservatively) the dispersion in $\log_{10}g$ for each bin, and *bottom*: the empirical fit to their data (equation 5) with 0.05 dex error bars, based on fits to galaxy rotation curves (Li et al. 2018). It is clear that α should lie in the range (0.4–1), with $\alpha < 0.1$ strongly excluded.

This RAR-inspired interpolating function is rather similar to the ‘simple’ form $v_s = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{x}}$ (Famaey & Binney 2005) for $x \lesssim 10$ (e.g. fig. 1 of Chae et al. 2019). The main difference is that the exponential term causes much faster convergence to Newtonian dynamics at large x , minimizing the impact on the Solar System.

The RAR has an intrinsic scatter < 0.05 dex once observational uncertainties are taken into account (Li et al. 2018). We take this as the uncertainty in the mean relation, i.e. in the value of v . Fig. 1 shows the binned RAR data (Lelli et al. 2017) and their fit (equation 5) with 0.05 dex error bars for $g_N = (0.01\text{--}100)a_0$, the approximate range of the data set. Any viable modification to gravity should have an interpolating function that passes through this region. We also show v_{GD} for different values of the parameter α (equation 4). To match constraints from galaxy rotation curves, $\alpha = 0.4\text{--}1$ since $a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}$ is fixed by the relation between galaxy baryonic masses and flatline rotation curve levels (Begeman et al. 1991).

3 SOLAR SYSTEM CONSTRAINT

Equation (1) indicates that the MOND radius of the Sun is $r_M = 7000$ astronomical units (7 kAU), implying that the whole Solar

System lies well inside r_M and is not much affected by MOND. However, the vastly superior data quality within the Solar System makes it important to explore what the GD model predicts for $g_N \gg a_0$. We focus on spacecraft tracking data at heliocentric distances of $\lesssim 10$ AU. Planets in this region should feel an extra Sunwards acceleration of αa_0 (equation 3). This aspect of the dipole model was also discussed in section 4.1 of Hajdukovic (2020) and in its abstract, where it was claimed that such a small effect (estimated at $6 \times 10^{-11} \text{ m s}^{-2}$) cannot be detected in the Solar System. We will see that this is incorrect.

3.1 Order of magnitude estimate

After some observing duration t , the anomalous acceleration of αa_0 implies a position drift of $\approx \alpha a_0 t^2/2$ starting from the same initial position and velocity. For $\alpha = 0.4$, the expected deviation from Keplerian motion is 600 km after 5 yr for a planet like Saturn with a much longer orbital period.

Iorio (2019) first pointed out that this casts serious doubt on the GD model since radio tracking data for the Cassini spacecraft have an accuracy of 32 m, but we have not detected such large deviations from its conventionally calculated trajectory (table 11 of Viswanathan et al. 2017).

3.2 Epicyclic approximation

We now consider how a planet’s trajectory would be affected by a small non-Newtonian force. Our derivation will be similar to that in Banik & Kroupa (2019).

Consider a planet on a near-circular orbit around the Sun at heliocentric radius r_p . Its Keplerian orbital frequency $\Omega = \sqrt{GM_\odot/r_p^3}$. Small radial perturbations r can be understood using the epicyclic approximation $\ddot{r} = -\Omega^2 r$.¹ Since this is linear in r , we may subtract whatever epicyclic oscillations the planet is undergoing and concentrate on the evolution of any remaining anomaly δr . This anomaly is sensitive to any additional non-Keplerian forces acting on the planet. Here, we focus on the extra Sunwards acceleration of αa_0 in the GD model.

$$\delta \ddot{r} = -\Omega^2 \delta r - \alpha a_0. \quad (6)$$

This should be solved subject to the condition $\delta r = \dot{\delta r} = 0$ at some initial time $t = 0$ when precise observations of the planet start. There is no way to know if its position and velocity have been altered by non-Keplerian forces at earlier times. Such forces are relevant only in so far as they cause the planet to deviate from the Keplerian trajectory corresponding to its osculating orbital elements at $t = 0$. Thus, the relevant solution to equation (6) is

$$\delta r = x_0 (\cos \phi - 1), \text{ where } \phi \equiv \Omega t \text{ and} \quad (7)$$

$$x_0 = \frac{\alpha a_0}{\Omega^2} = \frac{\alpha a_0 r_p^3}{GM_\odot}. \quad (8)$$

For Saturn orbiting at $r_{\text{Sat}} = 9.58 \text{ AU}$, $x_0 = 1065 \text{ km}$.

The GD model yields an extra Sunwards force, implying angular momentum is still conserved. Thus, the angular velocity $\propto (r_p + \delta r)^{-2}$, leading to a perturbation δp parallel to the circular orbital velocity. This is governed by

$$\delta \dot{p} = -2\Omega \delta r. \quad (9)$$

¹Epicyclic and orbital frequencies are the same only for an inverse square central force.

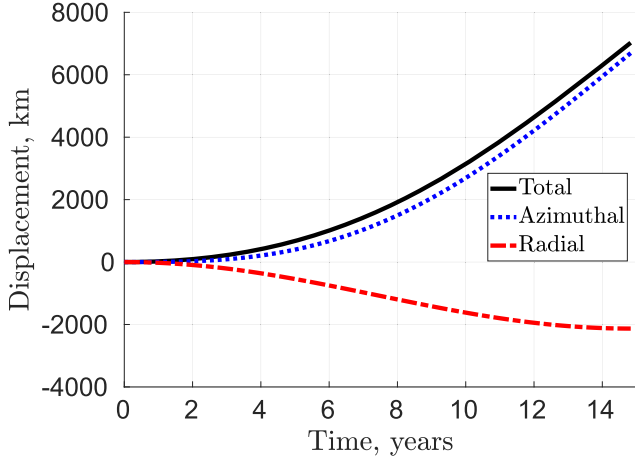


Figure 2. The difference between Newtonian and GD predictions (former subtracted) for the position of Saturn if $\alpha = 0.4$, the minimum needed for consistency with galaxy rotation curves (Fig. 1). The displacements are $\alpha\alpha$. Azimuthal displacements are positive in the direction Saturn rotates around the Sun. The much smaller orbit of the Earth is much less affected by GDs (equation 8). Thus, the difference between the models in the Earth–Saturn range is approximately that in Saturn’s heliocentric distance.

For reasons just explained, we also require $\delta p = \delta \dot{p} = 0$ at $t = 0$, though the latter is guaranteed if $\delta r = 0$ then. Thus, the relevant solution to equation (9) is

$$\delta p = 2x_0 (\phi - \sin \phi). \quad (10)$$

The orbital period of Saturn is ≈ 30 yr and the Cassini spacecraft orbited it for almost half this time. The GD-induced displacement of Saturn when $\phi = \pi$ is

$$\sqrt{\delta r^2 + \delta p^2} = 2x_0 \sqrt{1 + \pi^2} = 7025 \text{ km}. \quad (11)$$

The displacement at earlier times is illustrated in Fig. 2.

3.3 Comparison with observations

The latest planetary ephemerides for Saturn are accurate to 32 m (table 11 of Viswanathan et al. 2017) thanks to radio tracking data from the Cassini spacecraft, which orbited Saturn for 13 yr (Matson 1992). The null detection of deviations from its conventionally calculated trajectory implies $\alpha \lesssim 10^{-5}$. Since $\alpha \lesssim 0.4$ violates galaxy rotation curve data (Fig. 1), the GD model must be discarded in its present form. This conclusion was previously reached by Iorio (2019) based on the rate of perihelion precession, but frequent radio tracking of Cassini provides much more information than just this time-averaged quantity.

Our results completely rule out MOND interpolating functions of the form in equation (4). This form is fundamental to the whole GD approach (section 2 of Hajdukovic 2014b). In particular, the tanh function is derived directly from the partition function of the dipoles (section 4.2 of Hajdukovic 2020). In the remainder of this contribution, we explore the GD model in more detail to check if it contains features that might weaken this falsification.

3.4 Effects of other planets and their dipole haloes

In the Solar System, the GDs would be fully saturated as $g_N \gg a_0$. This approximation – also made in section 5.1 of Hajdukovic (2020) – implies that the polarization density \mathbf{P} has its maximum

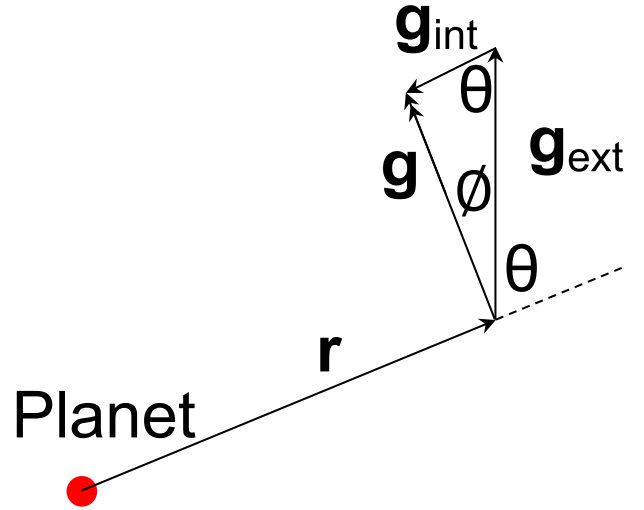


Figure 3. The total gravitational field \mathbf{g} at planet-centric position \mathbf{r} is the vector sum of the external Solar gravity \mathbf{g}_{ext} and the planetary gravity \mathbf{g}_{int} , which deflects the direction of \mathbf{g} by a small angle ϕ in the region where the Solar gravity dominates ($g_{\text{int}} \ll g_{\text{ext}}$). θ is the angle between \mathbf{g}_{ext} and \mathbf{r} .

strength of P_{max} and is aligned with the gravitational field \mathbf{g} . Thus, the QV effective density

$$\rho_{\text{qv}} = -\nabla \cdot \mathbf{P}, \text{ where } \mathbf{P} = P_{\text{max}} \hat{\mathbf{g}} (g \gg a_0). \quad (12)$$

We use the convention that $v \equiv |\mathbf{v}|$ for any vector \mathbf{v} and $\hat{\mathbf{v}}$ is the unit vector in the same direction. In the Solar System, GDs have only a small effect on \mathbf{g} , so $\hat{\mathbf{g}} \approx \hat{\mathbf{g}}_N$. To get an extra Sunwards acceleration of αa_0 , we must have that

$$P_{\text{max}} = \frac{\alpha a_0}{4\pi G}. \quad (13)$$

We now find the total mass of the GD halo associated with a planet of mass M_p embedded in an external gravitational field \mathbf{g}_{ext} from the Sun. The Solar gravity is assumed to be uniform over the whole planetary halo, an assumption we will justify later.

We use spherical polar coordinates (r, θ) centred on the planet with axis parallel to $\hat{\mathbf{g}}_{\text{ext}}$, i.e. towards the Sun. Applying the divergence theorem to a spherical region of radius r centred on the planet, we get that the volume integral

$$\int \rho_{\text{qv}} dV = -P_{\text{max}} \int_0^\pi (\hat{\mathbf{g}} \cdot \hat{\mathbf{r}}) 2\pi r^2 \sin \theta d\theta. \quad (14)$$

We can simplify this by taking r to be sufficiently large such that the Solar gravity \mathbf{g}_{ext} dominates over the planetary gravity $\mathbf{g}_{\text{int}} = -GM_p \hat{\mathbf{r}}/r^2$. Without the planetary gravity, $\hat{\mathbf{g}} = \hat{\mathbf{g}}_{\text{ext}}$ and so $\hat{\mathbf{g}} \cdot \hat{\mathbf{r}} = \cos \theta$, implying $\int \rho_{\text{qv}} dV = 0$. Thus, we need the first-order perturbation to $\hat{\mathbf{g}}$ caused by \mathbf{g}_{int} . The situation is illustrated in Fig. 3.

As $g_{\text{int}} \ll g_{\text{ext}}$, the planetary gravity rotates $\hat{\mathbf{g}}$ towards the planet by a small angle

$$\phi \approx \frac{g_{\text{int}} \sin \theta}{g_{\text{ext}}} = \frac{GM_p \sin \theta}{r^2 g_{\text{ext}}}. \quad (15)$$

Since $\phi \ll 1$, we get that

$$\hat{\mathbf{g}} \cdot \hat{\mathbf{r}} \equiv \cos(\theta + \phi) \approx \cos \theta - \frac{GM_p \sin^2 \theta}{r^2 g_{\text{ext}}}. \quad (16)$$

Substituting this into equation (14) and using $\int_0^\pi \sin^3 \theta d\theta = 4/3$, we obtain a planetary QV halo mass of

$$M_{\text{halo,p}} = \frac{8\pi G M_p P_{\text{max}}}{3g_{\text{ext}}}. \quad (17)$$

This result is independent of r , implying that the GD halo has a finite mass. This is similar to how the MOND phantom dark matter halo is truncated in the presence of an external field (e.g. Wu et al. 2008). The r -independence of our result shows that the effective mass does not lie in the region where $g_{\text{int}} \ll g_{\text{ext}}$, so it must lie closer to the planet in the region where its gravity dominates. This means that the planetary GD halo has an approximate size of r_h , defined by the distance where $g_{\text{int}} = g_{\text{ext}}$. A planet at heliocentric distance r_p has

$$r_h = r_p \sqrt{\frac{M_p}{M_\odot}}. \quad (18)$$

For Jupiter, $r_h = 0.16$ AU. As other Solar System planets have even less mass, their GD haloes are concentrated within a very small fraction of their orbital radius. This justifies our assumption that the Solar gravity can be treated as uniform over the planetary halo.

Substituting in equation (13) and the fact that $g_{\text{ext}} = GM_\odot/r_p^2$, equation (17) can be written as

$$M_{\text{halo,p}} = \frac{2\alpha a_0 M_p r_p^2}{3GM_\odot} = \frac{2}{3}\alpha M_p \left(\frac{r_p}{r_M}\right)^2. \quad (19)$$

The Solar GD halo interior to the planetary orbit has an effective mass of

$$M_{\text{halo},\odot}(< r_p) = \frac{\alpha a_0 r_p^2}{G} = \alpha M_\odot \left(\frac{r_p}{r_M}\right)^2. \quad (20)$$

As the Solar dipole halo is spherically symmetric, a planet is not affected by ρ_{qv} outside its orbit (though see Section 3.6). We can combine equations (19) and (20) to find the ratio between the Jovian effective halo mass and that of the Solar halo interior to the orbit of Saturn.

$$\frac{M_{\text{halo,Jup}}}{M_{\text{halo},\odot}(< r_{\text{Sat}})} = \frac{2M_{\text{Jup}}}{3M_\odot} \left(\frac{r_{\text{Jup}}}{r_{\text{Sat}}}\right)^2 = 1.9 \times 10^{-4}. \quad (21)$$

Clearly, the Solar dipole halo is several thousand times more important to Saturn than the dipole haloes of other planets.

This result can be understood by approximating that the planetary gravity is completely dominant inside the region $r < r_h$ and completely negligible outside, such that $\rho_{\text{qv}} \neq 0$ only if $r < r_h$. In this case, we would get that

$$M_{\text{halo,p}} \approx 4\pi r_h^2 P_{\text{max}} = \frac{\alpha a_0 M_p r_p^2}{GM_\odot} = \alpha M_p \left(\frac{r_p}{r_M}\right)^2. \quad (22)$$

This quick estimate agrees rather well with our result in equation (19). The halo mass $\propto r_h^2$ because the total amount of GDs scales with volume but it is their divergence which is relevant, leading to an additional $1/r_h$ geometric factor. This is why the GD theory predicts a fixed extra acceleration towards a point mass in the saturated regime (equation 3). As a result, Jupiter's dipole halo has an effective mass $\approx (0.16/9.6)^2 \times$ smaller than that in the Sun's dipole halo interior to the 9.6 AU radius of Saturn's orbit.

Therefore, the anomalous Sunwards acceleration of Saturn is almost entirely determined by the Solar GD halo. Neglecting the haloes of other planets leads to a fractional uncertainty of order 10^{-4} , while neglecting its own halo is justified by the fact that a planet should not be accelerated by its own halo. Moreover, increasing the

effective mass of Jupiter (or any other planet) yields an extra force towards the planet rather than towards the Sun. Hence, an extra Sunwards acceleration of αa_0 cannot be cancelled out by arbitrarily adjusting the effective masses of other planets.

The only possible exception is if we adjust the masses of the terrestrial planets, as one could argue that these are in a similar direction to the Sun as perceived from Saturn. Equation (20) implies that we would need to reduce the mass of e.g. Venus by $\Delta M = 0.25M_\oplus$, almost $5 \times$ the mass of Mercury. We would need a different ΔM in order to correctly 'tune' the orbit of a different gas giant since $M_{\text{halo},\odot}(< r_p) \propto r_p^2$. In any case, such large values of ΔM greatly exceed observational uncertainties because all known Solar System planets have rather well-known masses thanks to data from close spacecraft fly-bys. This also applies to the Sun – the Solar gravitational parameter GM_\odot has been measured to an accuracy of $2.5 \times 10^{-4} GM_\oplus$ (Pitjeva 2015).

Hajdukovic (2019) claim that including the gravity of dipoles that 'do not belong to any individual halo' might alleviate the above issues. Since unpolarized (randomly aligned) GDs have no divergence and thus contribute nothing to ρ_{qv} (equation 12), this might refer to regions where the gravity from a planet is comparable to that from the Sun. It is precisely these regions which we have just analysed in detail. There is another situation where it is difficult to identify the dominant object – at heliocentric distances of several kAU, the Galactic gravity becomes important. In Section 3.6, we consider the possible impact of GDs in this region on the orbits of Solar System planets.

3.5 Additional planets in the outer Solar System

An undiscovered ninth planet (P9) several hundred AU from the Sun (Batygin & Brown 2016) would raise tides on the known Solar System, affecting planetary orbits. The tidal effect of P9 on the Sun–Saturn relative acceleration is

$$g_{\text{tide}} \approx \frac{GM_9 r_{\text{Sat}}}{r_9^3}, \quad (23)$$

where r_9 is the present heliocentric distance of P9. For g_{tide} to be strong enough to correctly tune the orbit of Saturn, it must be comparable to αa_0 , implying that

$$M_9 \approx \frac{\alpha a_0 r_9^3}{Gr_{\text{Sat}}} = 281M_\oplus \left(\frac{r_9}{100 \text{ au}}\right)^3. \quad (24)$$

However, the proposed P9 has a mass of only $M_9 \approx 10M_\oplus$ and is more distant ($r_9 \gtrsim 200$ AU; Batygin & Brown 2016). In fact, the latest analysis indicates that P9 must be $\gtrsim 370$ AU away if $M_9 = 10M_\oplus$ (section 2.6 of Batygin et al. 2019). Such a distant planet should create only a very small tidal effect on the known Solar System.

In the GD model, part of the effective mass of P9 would come from its dipole halo. However, equation (22) shows that

$$\frac{M_{\text{halo,p}}}{M_p} = \frac{\alpha a_0 r_p^2}{GM_\odot} = 5.4 \times 10^{-5} \left(\frac{r_p}{100 \text{ au}}\right)^2 \left(\frac{\alpha}{0.4}\right). \quad (25)$$

Moreover, the M_9 estimate in Batygin & Brown (2016) should already include its dipole halo since it is based on how P9 perturbs the orbits of distant Kuiper Belt Objects.

Even if tides from P9 could affect the Sun–Saturn relative acceleration by αa_0 , P9 would not always pull Saturn directly away from the Sun over a 15 yr period. Cassini tracking data would easily reveal any lateral force of order αa_0 maintained for a few years

(Section 3.1). Moreover, extreme fine-tuning would be required for P9 to have the right tidal effect on the orbit of Saturn (equation 24).

The effect would be different on other planets as it would scale with the size of their orbit. However, the GD model predicts that they too should feel an extra Sunwards acceleration of αa_0 due to the Solar dipole halo interior to their orbit (equation 3). This means that different P9 masses are required to counter this effect for different planets. Consequently, the possible existence of P9 cannot reconcile the GD model with observations.

3.6 Effect of the Galactic gravity

The Solar dipole halo exterior to the orbit of Saturn does not affect its dynamics because the halo is spherically symmetric. This symmetry eventually breaks down to mere axisymmetry at distances $\gtrsim 5$ kAU due to the Galactic gravitational field. As a result, ρ_{qv} in this region can raise tides on the Solar System. Since the Galactic gravity on the Sun is of order a_0 (e.g. section 3.6 of Banik & Zhao 2018), we expect order unity deviations from Newtonian dynamics at the Sun's MOND radius of 7 kAU (equation 1). In other words, we expect tides from the non-spherical ρ_{qv} distribution to have a strength of order a_0 at this distance. Given that Saturn is < 10 AU from the Sun, the Sun–Saturn relative acceleration should only be affected by $\lesssim 10^{-3} a_0$.

Even if these tides were much stronger, tides raised by such distant ρ_{qv} cannot always cause a Sun–Saturn repulsion while Saturn completes half an orbit. This is because the tidal field due to ρ_{qv} outside the Solar System must be divergence free within the Solar System. Repulsive tides can be maintained around half a Saturnian orbit only if the ρ_{qv} distribution is axisymmetric and the symmetry axis aligns with the orbital pole of Saturn. However, there is a 95.5° angle between the North Ecliptic Pole and the direction towards the Galactic Centre. Therefore, tides raised by the non-spherical ρ_{qv} distribution several kAU from the Sun cannot cancel a constant extra Sunwards acceleration of order a_0 . Moreover, tidal accelerations scale with the size of the planetary orbit while the extra Sunwards acceleration in the GD model is the same for all planets, making it infeasible for tides to consistently cancel out this effect.

4 CONCLUSIONS

The GD theory (Hajdukovic 2010) is similar to MOND with an interpolating function given by equation (4). It seeks to explain the MOND phenomenology by postulating that antimatter has a negative gravitational mass and so falls upwards in the terrestrial gravitational field. This should be directly testable in the next few years.

The model also predicts that Solar System planets experience an extra Sunwards acceleration of αa_0 , where the MOND parameter $a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}$ in order to match rotation curve constraints. These require $\alpha \gtrsim 0.4$ (Fig. 1), consistent with the estimate that $\alpha = 5/12$, and should certainly lie in the range 0.4–0.5 (Hajdukovic 2020). Starting from the same initial position and velocity, this extra acceleration would cause the position of Saturn to deviate from conventional expectations by $7000(\alpha/0.4)$ km over 15 yr (Section 3.2). The Cassini mission at Saturn (Matson 1992) has been tracked over roughly this time span to a precision of 32 m (Viswanathan et al. 2017). No significant deviations were found from its conventionally calculated trajectory, casting serious doubt on the GD model (Iorio 2019).

One possible complication is that in addition to the Solar dipole halo responsible for an extra Sunwards acceleration, each planet would have its own dipole halo (Hajdukovic 2019). However, we

showed that these haloes are truncated when the Solar gravity dominates over that of the planet (Section 3.4). Consequently, the Solar dipole halo interior to the orbit of Saturn contains vastly more effective mass than the dipole halo of Jupiter, or indeed any other planet. Even if the effective masses of other planets were arbitrarily adjusted or new ones invented, their time-varying direction as viewed from Saturn makes it impossible for them to precisely cancel out an extra Sunwards acceleration. While this may be possible for a time-averaged quantity like the rate of Saturn's perihelion advance, there would still be km-scale deviations from its conventional trajectory during the course of an individual orbit. Since Cassini tracking data provide several thousand range measurements over half its orbital period, such deviations should have been detected. Moreover, the Solar dipole halo interior to Saturn has an effective mass of $0.25 M_\oplus$, vastly exceeding observational uncertainties on the masses of Solar System bodies and the effective masses of their dipole haloes.

The Galactic gravity causes the Solar dipole halo to depart from spherical symmetry at distances of $\gtrsim 5$ kAU. However, ρ_{qv} in this region should have only a small tidal effect on the Solar System (Section 3.6). Additionally, the force would not always be aligned with the Sun–Saturn line while Saturn completes half an orbit.

An extra Sunwards acceleration of order a_0 is thus a strong prediction of the GD model for any planet in the Solar System. This prediction is robust to the possible presence of a ninth planet in the outer Solar System (Section 3.5). Since the anomalous acceleration should be the same for all planets, it cannot be consistently cancelled by tides raised by sources outside the Solar System.

Our work confirms the finding of Iorio (2019) that the GD model is falsified in its present form because it predicts unobserved effects within the Solar System that exceed observational upper limits by ≈ 5 orders of magnitude. This falsification applies to any MOND-like force law with interpolating function given by equation (4) – galactic rotation curve fits require $\alpha > 0.4$ while Solar System constraints imply $\alpha < 10^{-4}$ at very high significance. More generally, Solar System ephemerides rule out any theory that predicts an enhancement to gravity by $\gtrsim 10^{-5} a_0$ arbitrarily deep into the Newtonian regime. Even though MOND need not imply such an effect (ν can be 1 for high g_N), Hees et al. (2016) constrained its interpolating function based on the fact that the Galactic external field causes the Solar phantom dark matter halo to become non-spherical at large distances, imposing a tidal stress (Blanchet & Novak 2011).

We conclude that the GD model proposed by Hajdukovic (2020) is unable to satisfy both galaxy rotation curve and Solar System constraints. The impressive successes of Milgromian dynamics must then be explained in some other way and other solutions must be found to the cosmological constant problem.

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